

CTP431: Fundamentals of Computer Music

Fundamentals of Musical Acoustics



Graduate School of
Culture Technology

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Goals

- Understand the physics of musical tone generation
 - Underlying mathematics
 - Properties of waves
 - Physical attributes of sound

- Human perception of sound
 - Physiology of human ears
 - Perceptual attributes of sound

What is Sound?

- Vibration of air pressure that we can hear through ears

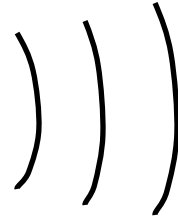
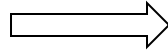
Physical perspective (waves)

Generation

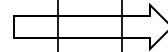


Vibration on an object

Propagation

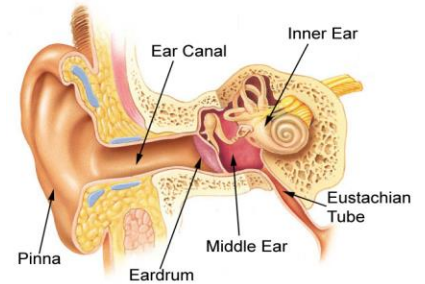


Traveling in air



Psychological perspective

Perception



Sensation of the air vibration through ears

Sound Generation



Sound Generation

Plucking, Striking
(momentary force)

Bowing, Blowing
(continuous force)

Playing

Force

**Musical
Instrument**

Sound

< 1D vibration >

String: Piano, Guitar, Violin

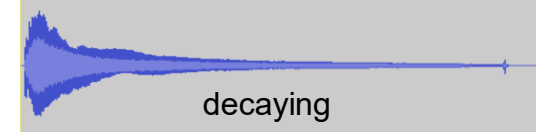
Pipe: Clarinet, Flute

< 2D vibration >

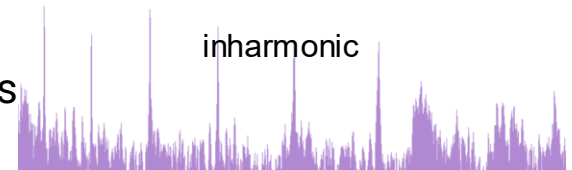
Membrane: Drum, Percussions

Bar: Marimba, Xylophone

Amplitude Envelope



Spectrum



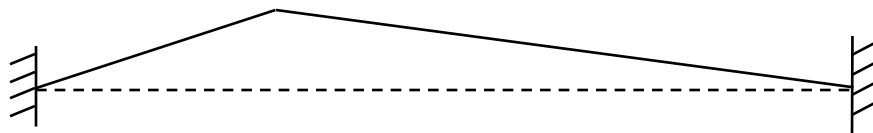
Oscillation: Plucked String



https://www.youtube.com/watch?v=_X72on6CSL0

Oscillation: Plucked String

- One-dimensional ideal string vibration



Mathematics

Wave Equation $K \frac{\partial^2 y}{\partial x^2} = \varepsilon \frac{\partial^2 y}{\partial t^2}$

Boundary Condition Fixed ends

Initial Condition Plucking at a position



Properties of Waves

Propagation

Reflection and Superposition

Modes and Standing Waves

Wave Equation

- For 1-D string (or other media) with displacement $y(x, t)$

Curvature
(how much the string is bended)

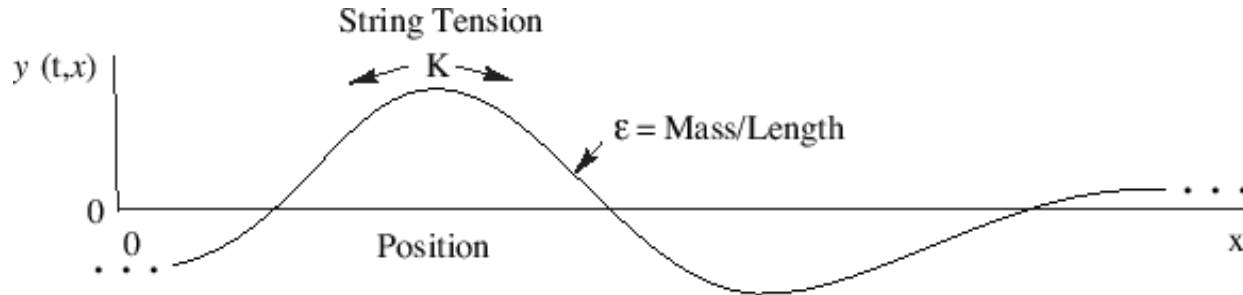
Stiffness
(how stiff the string is)

Acceleration

Linear mass density

$$F = ma$$
$$K \frac{\partial^2 y}{\partial x^2} = \epsilon \frac{\partial^2 y}{\partial t^2}$$

$c = \sqrt{\frac{K}{\epsilon}}$
Speed of Wave

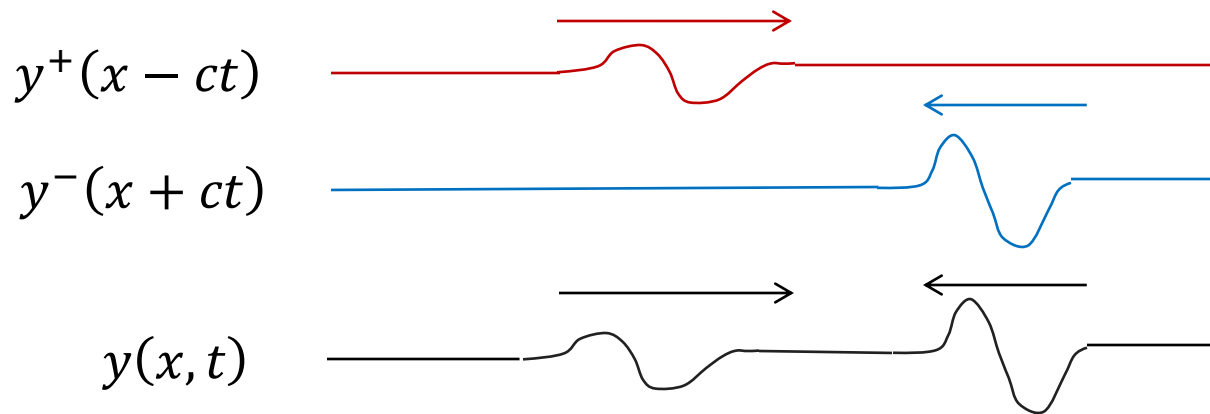


Wave Equation

- The general solution
 - The sum of **leftward-moving** y^- and **rightward-moving** y^+ traveling waves with a constant speed c

$$y(x, t) = y^+(x - ct) + y^-(x + ct)$$

- The shape of waveform does not change: set by the **initial conditions**

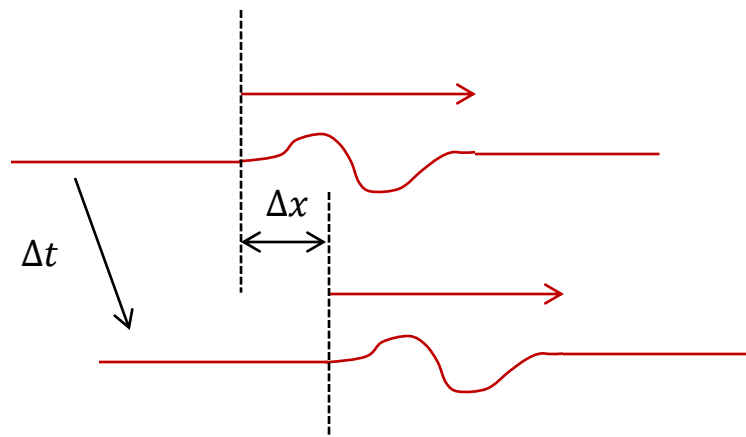


Wave Equation

- Why is it a traveling wave ?
 - Rightward-traveling wave case

$$y(x, t) = y^+(x - ct)$$

After Δt sec \longrightarrow $y(x, t + \Delta t) = y^+(x - c(t + \Delta t))$



$$= y^+(x - ct - c\Delta t)$$

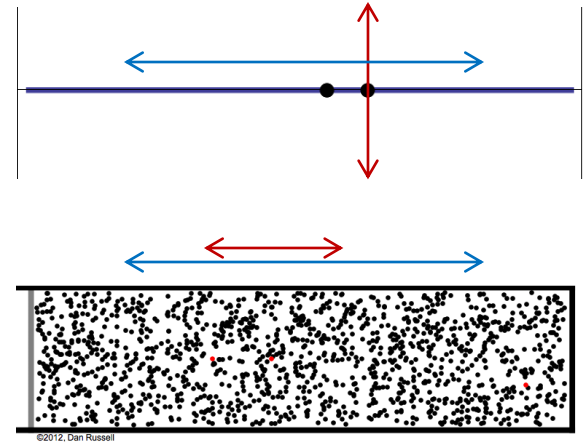
$$= y^+(x - ct - \Delta x) \longleftarrow c\Delta t = \Delta x$$

$$= y^+(x - \Delta x - ct)$$

$$= y(x - \Delta x, t) \longleftarrow \text{rightward-traveling by } \Delta x$$

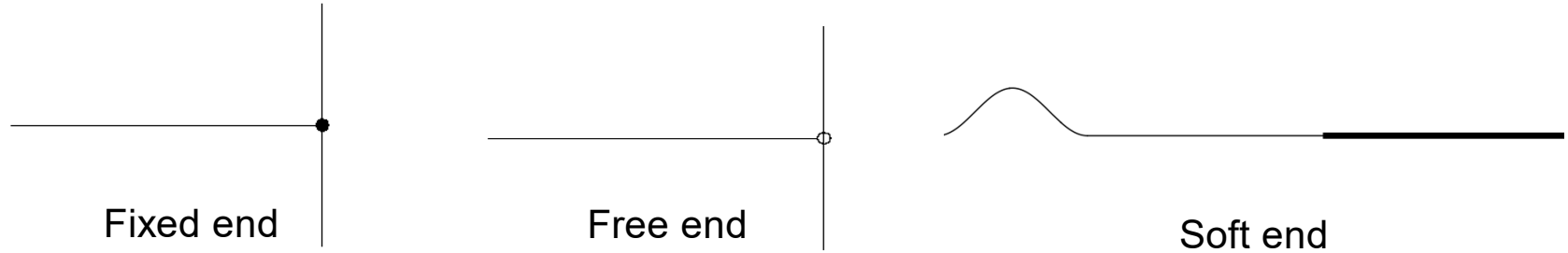
Wave Propagation

- The wave equation explains “wave propagation” in a medium
- Note that, while wave travels and transmits energy, the medium itself oscillates at the same position
 - Transverse wave
 - The wave travels through the medium in the perpendicular direction to the wave motion
 - Example: string, membrane, bar
 - Longitudinal wave
 - The wave travels through the medium in the same direction as the wave motion
 - Example: pipe (air)



Wave Reflection

- Wave is reflected when the impedance (Z) of medium changes



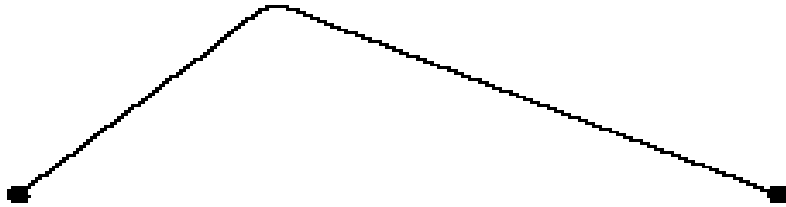
- Related to the **boundary conditions**
- Fixed end (Z is ∞): the incident wave is flipped
- Open end (Z is 0): the incident wave is mirrored
- Soft / hard end (Z is between 0 and ∞): the incident wave is partially flipped and transferred to the other side (e.g., hard end: bridge in string)



Source: <http://www.acs.psu.edu/drussell/Demos/reflect/reflect.html>

Wave Superposition

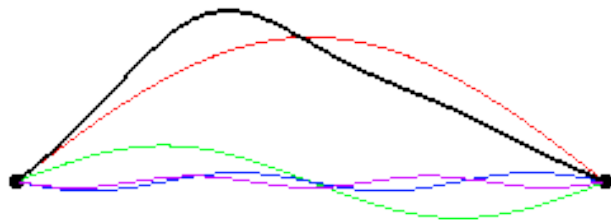
- Two traveling waves can form a single wave as a linear sum
- The superposition of the traveling wave and its reflections form this motion in the video



Source: <https://www.acs.psu.edu/drussell/Demos/string/Fixed.html>

Fourier Analysis

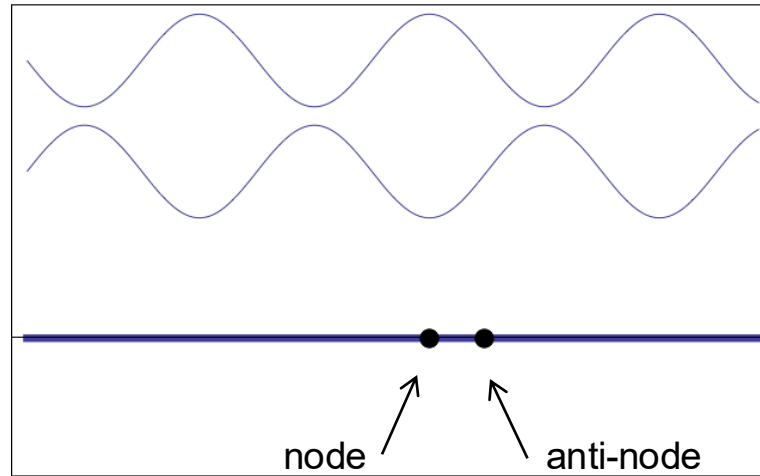
- Provides a way of representing the initial condition by the sum of sinusoids
 - The frequency (or period) of the sinusoids has an integer multiples relation
 - The amplitude and phase are determined by **Fourier series** as the waveform is a continuous-time and periodic signal



$$x(t) = \frac{1}{T} \left(A_0 + A_1 \cos\left(\frac{2\pi t}{T} + \phi_1\right) + A_2 \cos\left(\frac{2\pi 2t}{T} + \phi_2\right) + A_3 \cos\left(\frac{2\pi 3t}{T} + \phi_3\right) + \dots \right)$$

Standing Wave

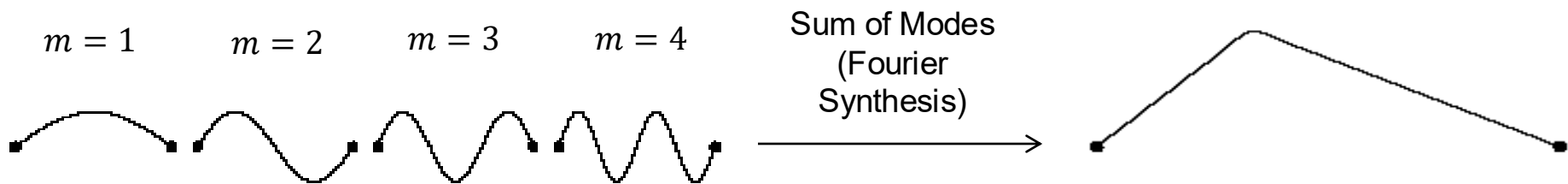
- Each of the sinusoidal shape travels to the left and right direction
- The sum of two travelling waves in opposite directions with the same frequency cancel or reinforce each other, creating a stationary oscillation



Source: <http://www.acs.psu.edu/drussell/Demos/superposition/superposition.html>

Mode

- Each of the standing waves form a mode with a sinusoidal oscillation
 - They are harmonic in frequency: **harmonic modes**
 - **Overtone**: higher modes other than the fundamental mode
 - **Partials**: indicates the frequency of each mode in the spectrum



$$\lambda = \frac{2L}{m}$$

Wave length

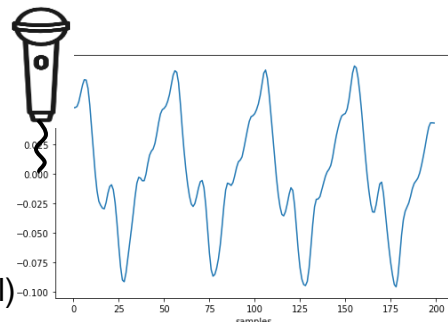
$$f = \frac{c}{\lambda} = \frac{m \cdot c}{2L}$$

Frequency

c : speed of wave
 L : the length of string
 m : mode number (1,2,3,...)

$$c = \sqrt{\frac{K}{\epsilon}}$$

speed of wave
(determined by the tension and material)



Damping

- A more practical setting of wave equation has a damping term

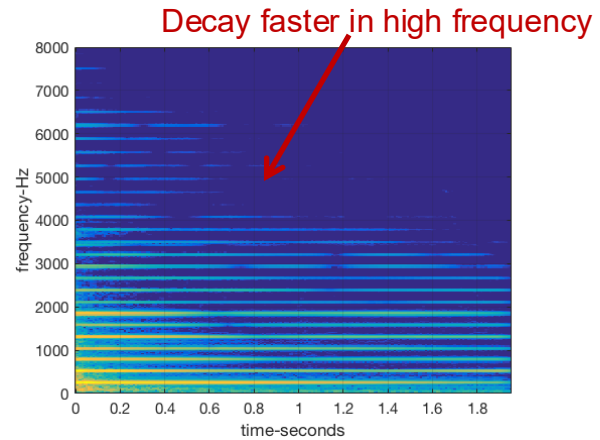
$$K \frac{\partial^2 y}{\partial x^2} = \varepsilon \frac{\partial^2 y}{\partial t^2} + \mu \frac{\partial y}{\partial t}$$

velocity

friction coefficient (resistance)

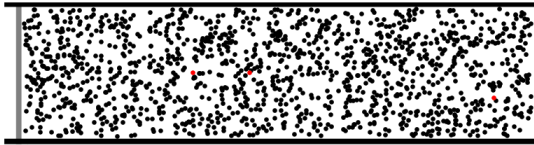
- The damping causes **exponential decaying** traveling wave
 - Higher frequency modes decay faster
 - The motion of plucked string becomes rounded over time

Spectrogram of a piano note

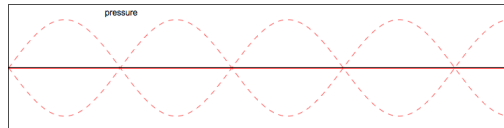
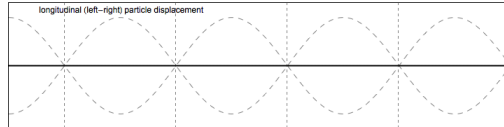


Pipes

- Longitudinal pressure wave to travel in the air column
 - Woodwind or brass instrument: flute, clarinet, trumpet
 - Blowing: continuous excitation
 - Clarinet: odd harmonics are generated



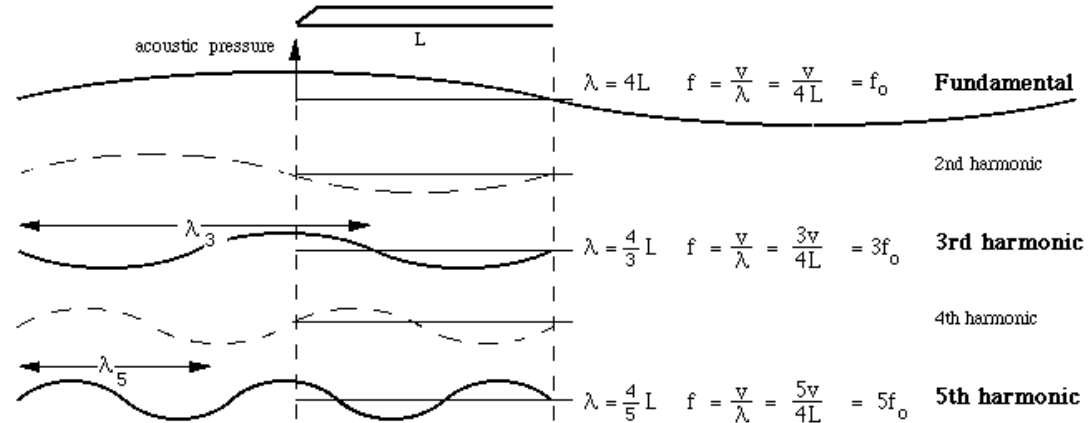
©2012, Dan Russell



Closed-end



Open-end



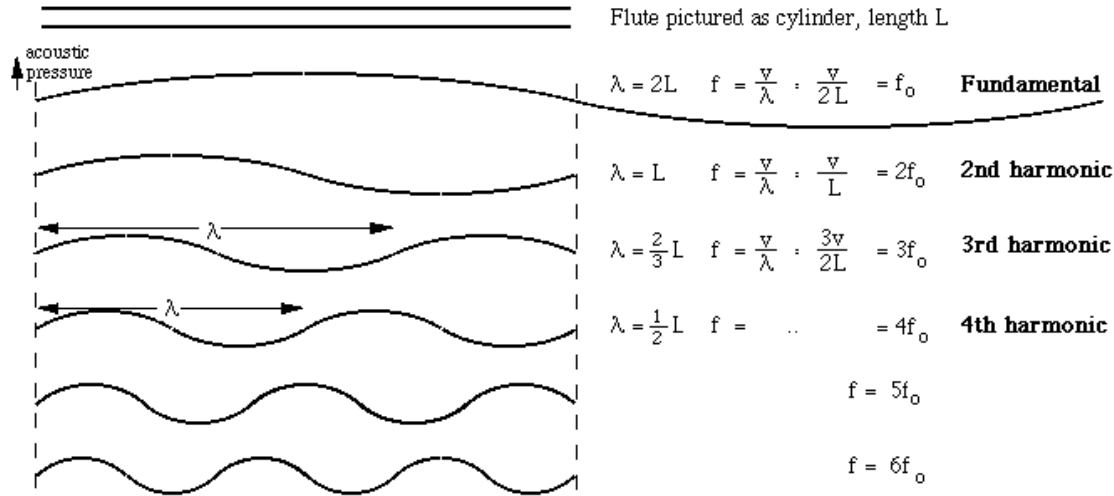
Pipes

- Longitudinal pressure wave to travel in the air column
 - Flute: All harmonic tones are generated

Open-end

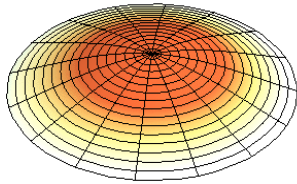


Open-end

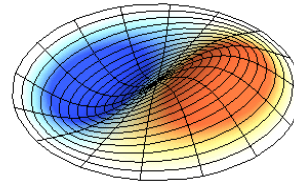


Membrane

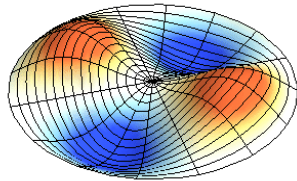
- 2D wave equation: $z(x, y, t)$
 - Drum, percussion
 - Boundary condition: determined by the shape of membrane
 - Circular harmonic oscillation → generate inharmonic tones



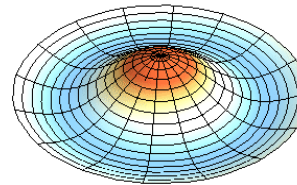
(0,1) Mode



(1,1) Mode



(2,1) Mode



(0,2) Mode

Chladni Plate

- Circular or rectangular vibrating plates that visualize 2D standing waves



<https://www.youtube.com/watch?v=1yaqUI4b974>

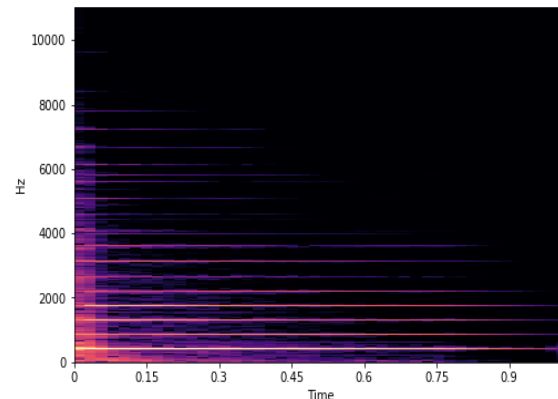
- Artistic examples
 - <https://www.youtube.com/watch?v=Q3oltpVa9fs>
 - <https://www.youtube.com/watch?v=qxdJ-ebNi14>

Summary

- Wave equation: travelling wave
 - Initial condition: by the human play on the musical instrument
 - Boundary condition: by the structure of the musical instrument
- Wave Properties
 - Propagation (traveling) by the wave equations
 - Reflection
 - Superposition
 - Standing waves
- Vibration modes
 - 1D: sinusoidal harmonic oscillations → harmonic tone
 - 2D: circular harmonic oscillations → complex inharmonic tone

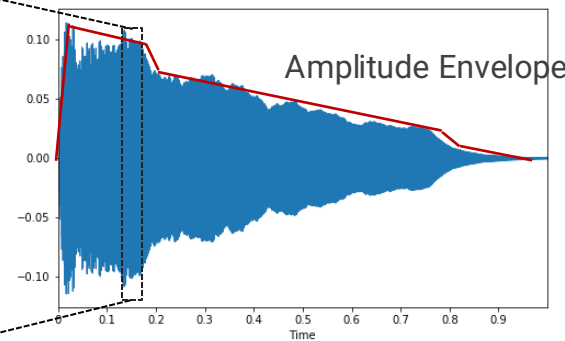
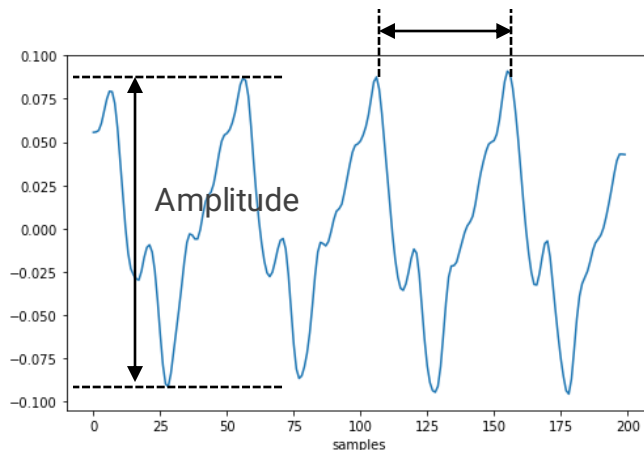
Physical Attributes of Sound

- Amplitude
- Frequency (Hz)
- Amplitude Envelope
- Duration
- Spectrum



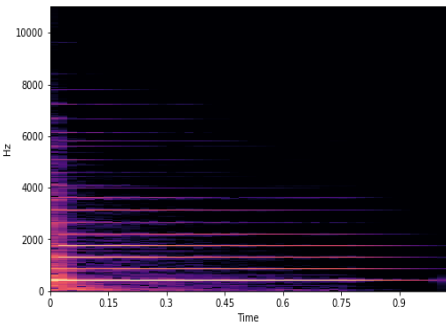
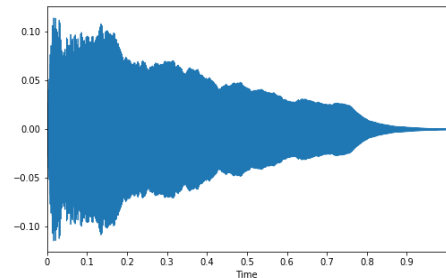
Spectrogram

Period = $1/(\text{fundamental frequency})$

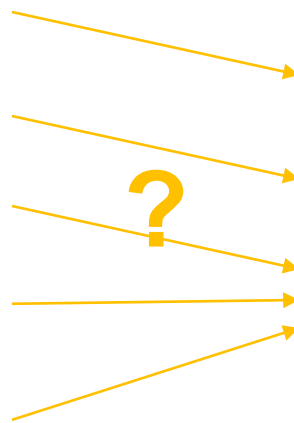


Waveform

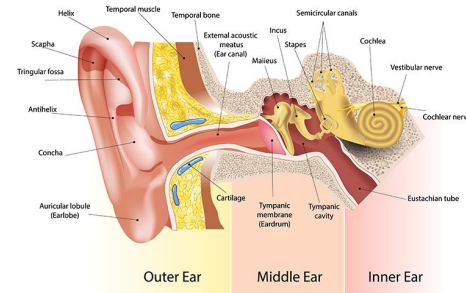
Perceptual Attributes of Sound



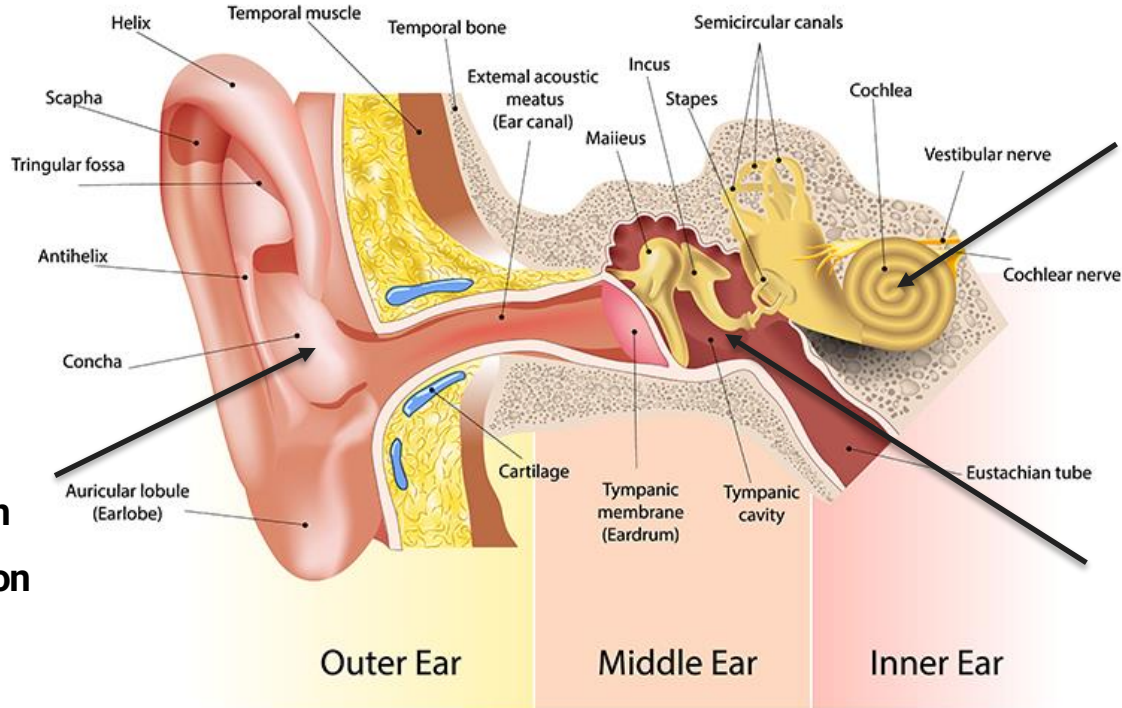
- Amplitude
- Frequency
- Envelope
- Duration
- Spectrum



- **Loudness**
- **Pitch**
- **Timbre**



Human Ear



✓ **Band-pass Filtering**

✓ **Electric nerve firing**

✓ **Impedance Matching**

✓ **Amplification**

✓ **Protection**

✓ **Localization**

✓ **Amplification**

Auditory Transduction in Human Ears

Auditory Transduction

<http://www.youtube.com/watch?v=PeTriGTENoc>

Auditory Transduction in Human Ears

- Multiple steps of transduction
 - Air vibration (outer ear) → Mechanical vibration (middle ear) → Fluid vibration (inner) → Nerve firing (brain)
- High sensitivity by the amplification
 - Outer ear and middle ear
- Tonotopic organization
 - Inner ear can be regarded as a bandpass filterbank

Outer Ear

- Pinnae

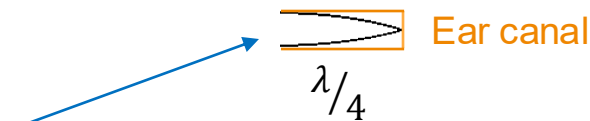
- Collect and localize sounds:

- <http://www.douglas-self.com/MUSEUM/COMMS/ear/ear.htm>

- **Localization:** related to recognize the direction of sound
 - head-related transfer function (HRTF)

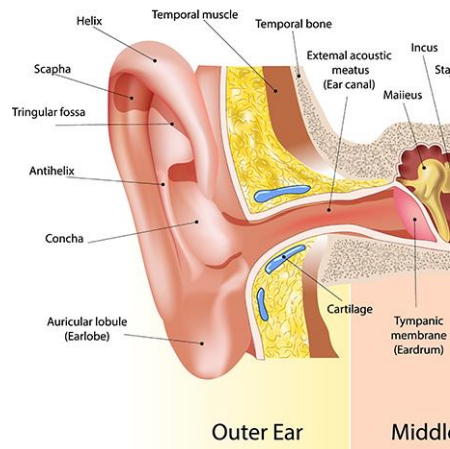
- Ear canal

- Protect ear drums
- Quarter-wave resonance: boost the vibration around 3kHz by 15-20 dB



- Ear drum

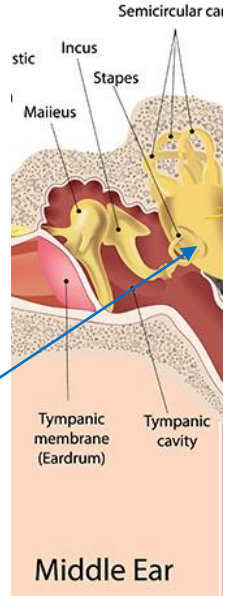
- Membrane that transduces air vibration to mechanical vibration
- Malleus (hammer) is attached to it



Middle Ear

- Ossicles

- malleus (hammer), incus (anvil) and stapes(stirrup)
- The smallest bones in human body
- **Impedance matching** between air pressure (outer) and fluid (inner)
 - Without ossicles, only about 1/30 of the sound energy would have been transferred to inner ear
- **Amplification:** the three bones work as a lever
 - The vibration from ear drum is transferred to a smaller size membrane (oval window) in the inner ear

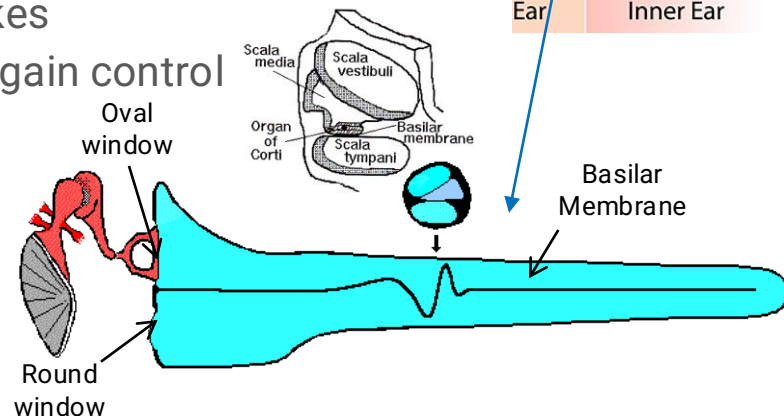
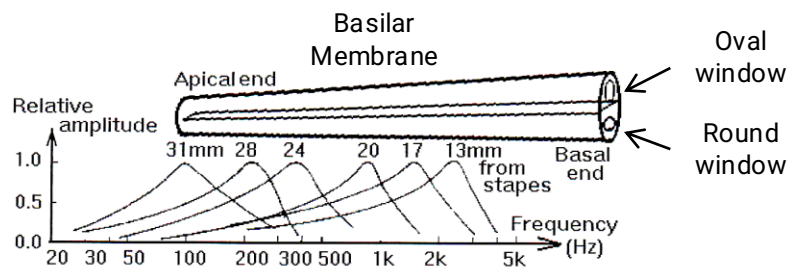


- Muscles

- Reduce the sound transmission in response to loud sounds

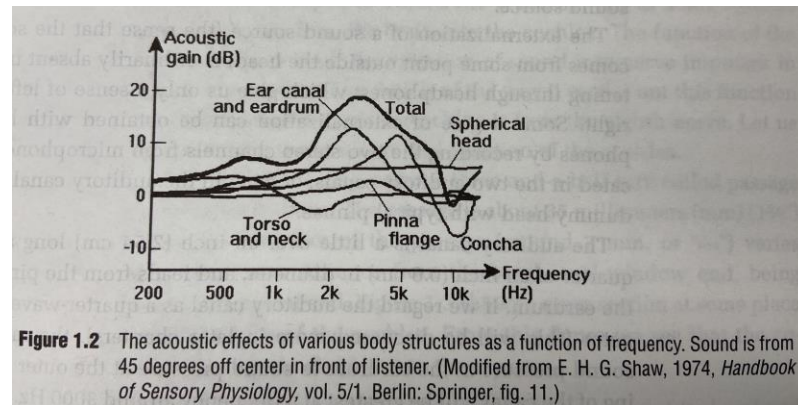
Inner Ears

- Cochlea: transduces fluid vibration to nerve firing
 - Basilar membrane
 - Vibrate at a different position depending on the frequency
 - Similar to band-pass filters
 - The bandwidth becomes wider as the frequency goes up
 - Organ of Corti
 - One row of inner hair-cell fire neural spikes
 - Three rows of outer hair-cell: automatic gain control



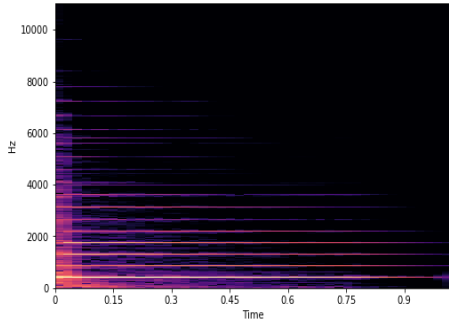
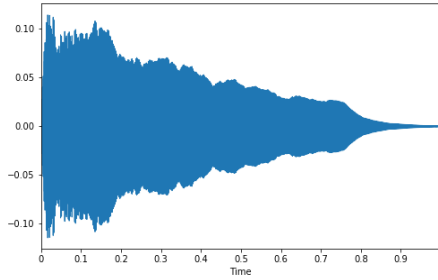
Sound Transformation in Ear

- Outer ear
 - HRTF: sound location-dependent filters
 - Ear canal: resonance around 3kHz by 15-20 dB
- Middle ear
 - Amplification (three bones)
- Inner ear
 - Basilar membrane: sub-band decomposition
 - Inner hair-cell: non-linear processing

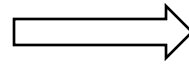


Source: P. R. Cook (Editor) Music, Cognition, and Computerized Sound (book)

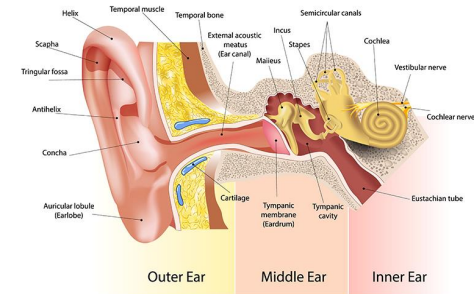
Perception of Sound



- Amplitude
- Frequency
- Envelope
- Duration
- Spectrum



- **Loudness**
- **Pitch**
- **Timbre**



Loudness Perception

- Subjective perception of sound intensity (the squared amplitude)
- Weber's law: $\Delta I \propto I$
 - Just Noticeable Difference (JND) of intensity (ΔI) is proportional to the intensity (I) at the moment
 - An exponential function satisfies this rule $\rightarrow \log(I)$ becomes linear

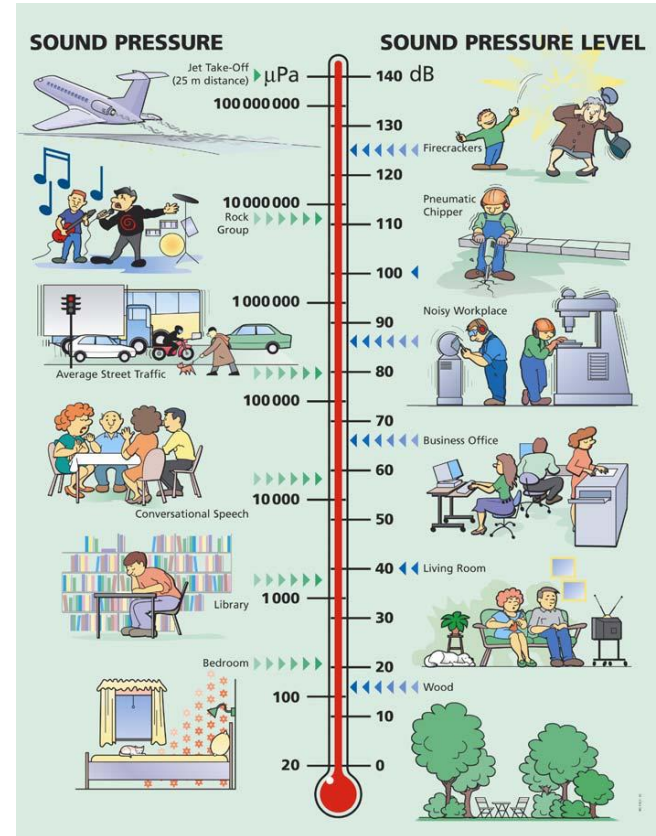


Loudness Perception

- Sound Pressure Level (SPL)
 - Physically measured sound intensity
 - Decibel scale: $20\log_{10}(P/P_0) = 10\log_{10}(I/I_0)$
 - The softest audible level: $P_0 = 2 \times 10^{-5} \text{ N/m}^2$



SPL meter

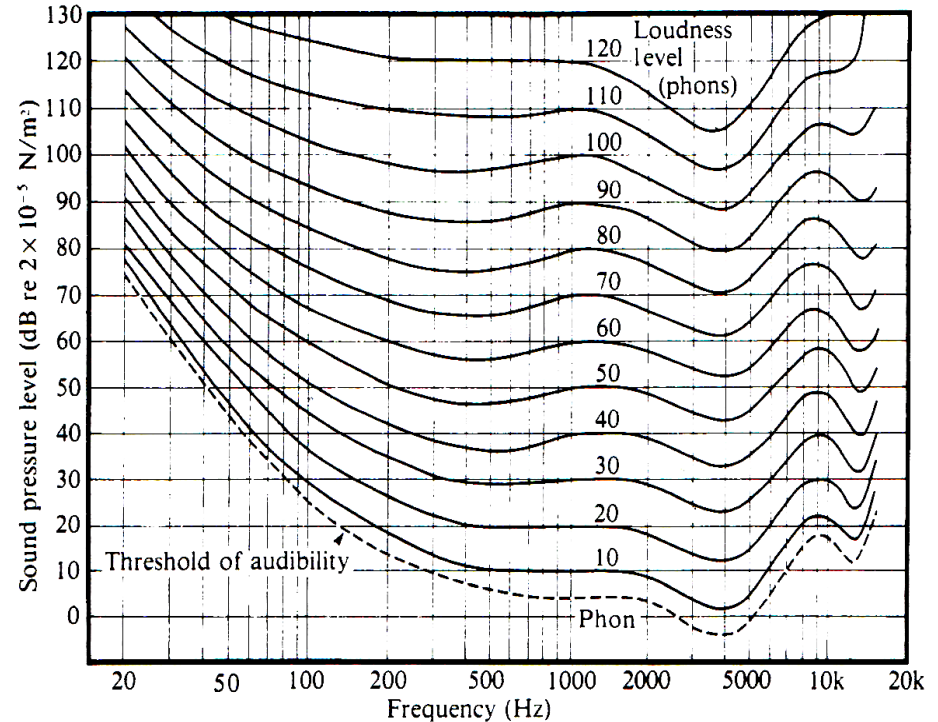


Source: http://www.audioholics.com/home-theater-connection/basic-home-theater-setup-guide/splmeter500x332.jpg/image_view_fullscreen

Source: [http://www.bksv.com/doc/sve/7.%20Audiology%20\(0210\).pdf](http://www.bksv.com/doc/sve/7.%20Audiology%20(0210).pdf)

Equal-Loudness Curves

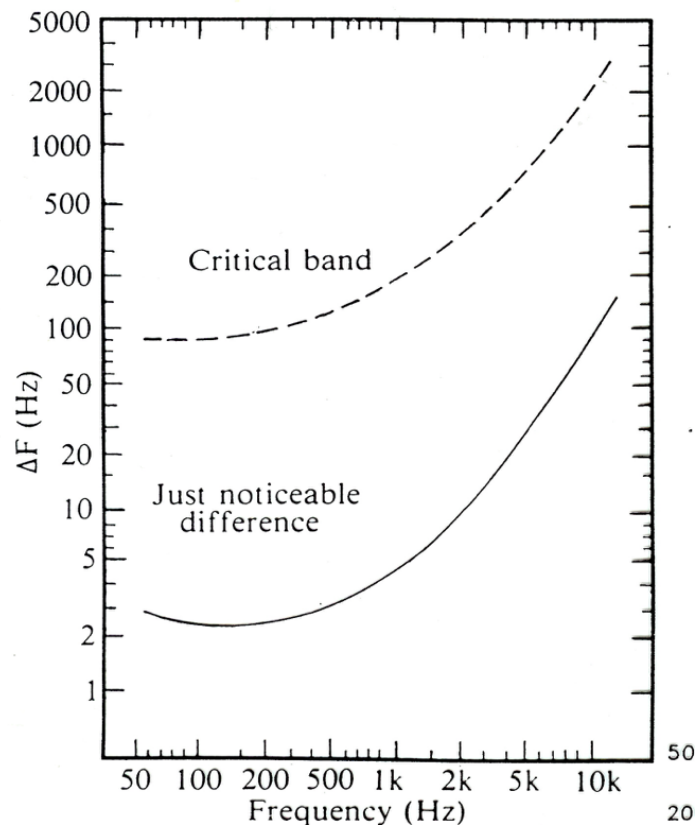
- Loudness depends on frequency (Fletcher and Munson, 1933)
 - 1kHz is used as a reference
 - Most sensitive to 2-5 kHz tones
 - Resonance in the ear canal
 - See the threshold of audibility
 - Dynamic ranges are different



Do your own test: <http://newt.phys.unsw.edu.au/jw/hearing.html>

Pitch Perception

- Audible pitch range
 - From 20Hz to 20kHz
 - Upper limits gradually decreases with age and also how much you are exposed to strong noises
- Pitch resolution
 - Just noticeable difference (JND) depends on not only the frequency but also the sound level and the duration of tone.
 - This is related to pitch scale

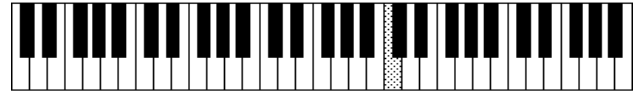


Pitch Scale

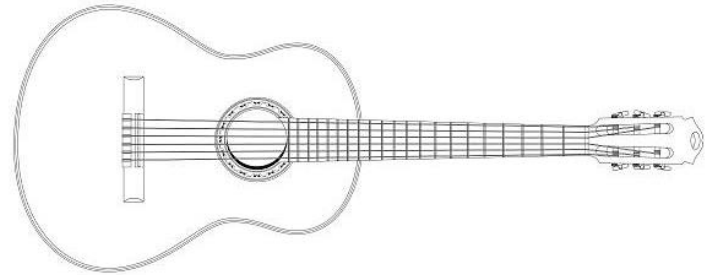
- Human ears are sensitive to frequency changes in a log scale
 - Physical location of basilar membrane
- Approximated pitch scale by subjective tests
 - Mel scale $m = 2595 \log_{10}(1 + f / 700)$
 - Based on pitch ratio of tones
 - Most popularly used for audio analysis
 - Bark scale $Bark = 13 \arctan(0.00075f) + 3.5 \arctan((f / 7500)^2)$
 - Critical band measurement by masking
 - Used in audio compression

Pitch Scale in Music

- Musical temperament
 - Determining the frequencies of all 12 notes within one octave
 - Tuning musical instruments



- Types
 - Pythagorean tuning
 - Just intonation
 - Equal temperament



Pitch Scale in Music

- Pythagorean tuning

- Based on the 3:2 ratio for perfect fifth interval

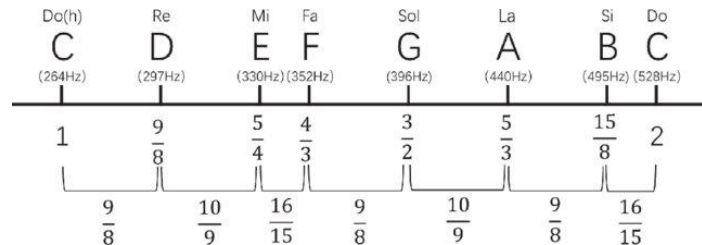
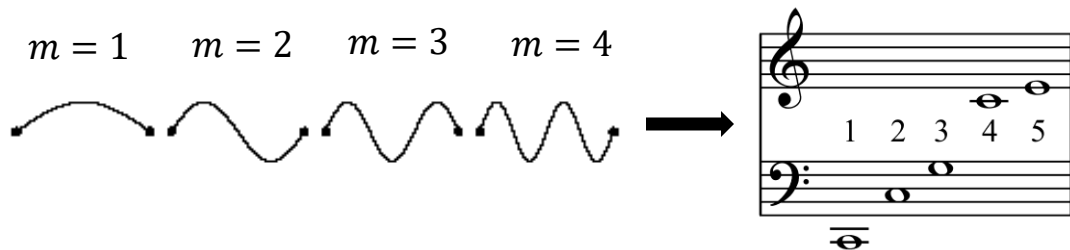
- Issue: “Wolf fifth” issue: $(\frac{3}{2})^{12} \neq 2^7$

- $C0 \rightarrow G0 \rightarrow D1 \rightarrow A1 \rightarrow E2 \rightarrow B2 \rightarrow F\#3 \rightarrow Db4 \rightarrow Ab4 \rightarrow Eb5 \rightarrow Bb5 \rightarrow F6 \rightarrow C7$

- Just intonation

- Based on harmonic series of a tone

- Issue: key transpose changes the temperament

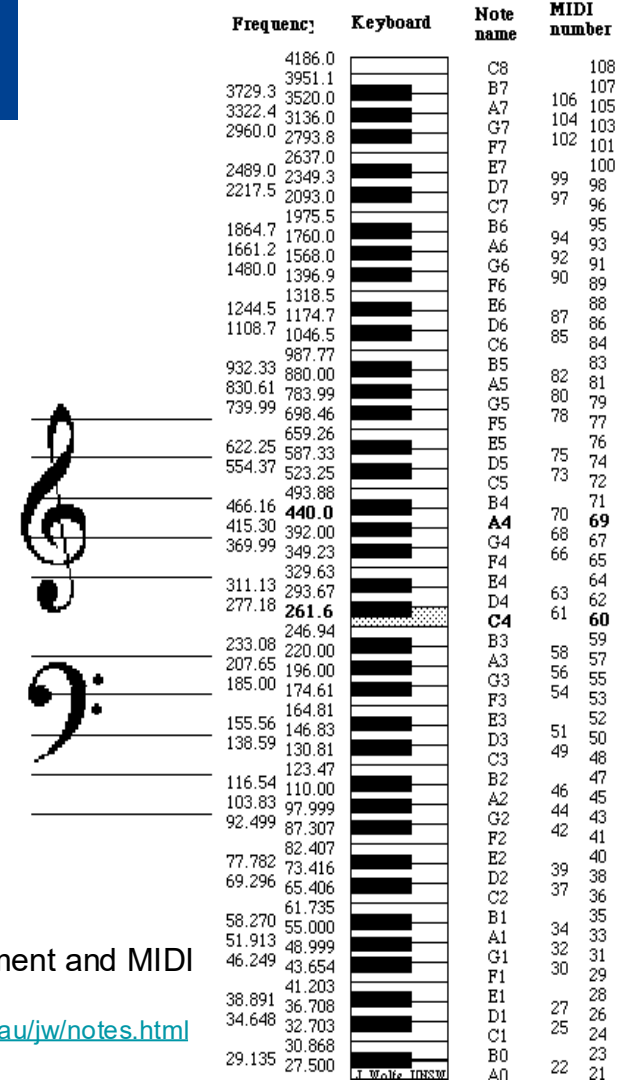


Pitch Scale in Music

- Equal temperament
 - 1: $2^{1/12}$ ratio between two adjacent notes
 - Music note (m) and frequency (f) in Hz

$$m = 12 \log_2\left(\frac{f}{440}\right) + 69, \quad f = 440 \cdot 2^{(m-69)/12}$$

- Invariant to key transpose



Equal temperament and MIDI

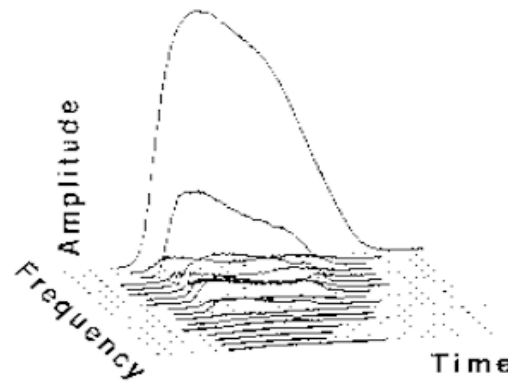
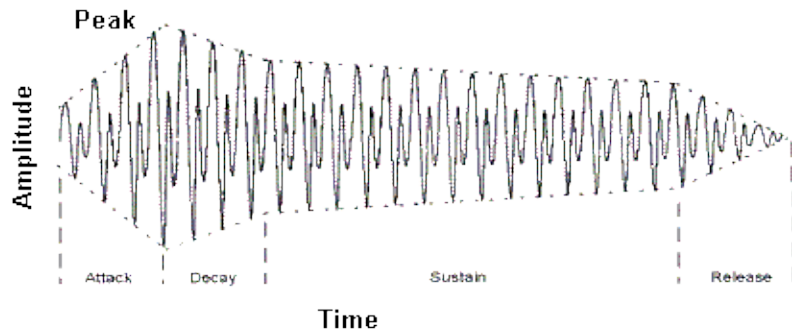
Source: <https://newt.phys.unsw.edu.au/jw/notes.html>

Timbre Perception

- Attribute of sensation by which a listener can judge two sounds having the same loudness and pitch are dissimilar (ANSI)
 - Tone color or quality that defines a particular sound
- Associated with classifying or identifying sound sources
 - Class: speech, piano, guitar
 - Identity: human voice
- Timbre is a multi-dimensional concept
 - Multiple physical attributes are associated with timbre
 - Timbre space: measuring timbre dimension based on perceptual similarity of different sounds

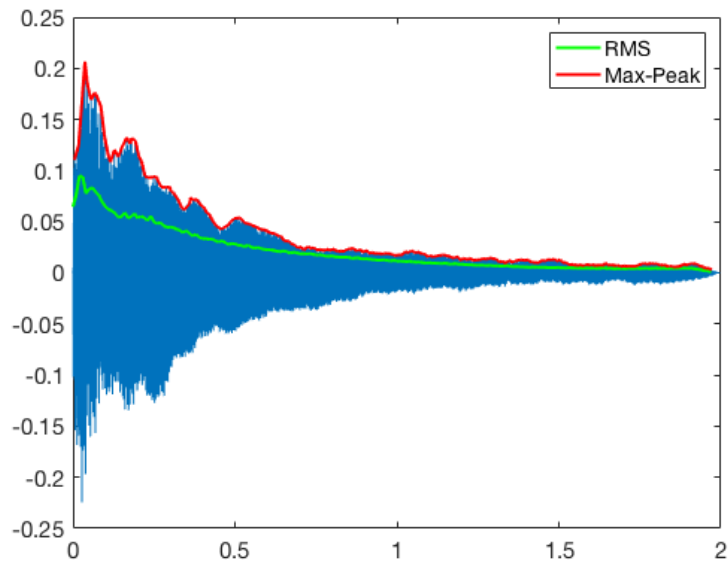
Physical Attributes in Timbre Perception

- Physical attributes that change timbre (Schouten, 1968)
 - Amplitude envelope
 - Spectral envelope
 - Changes of spectral envelope and fundamental frequency
 - Harmonicity: the range between tonal and noise-like character
 - The onset of a sound differing notably from the sustained vibration
 - Inharmonicity

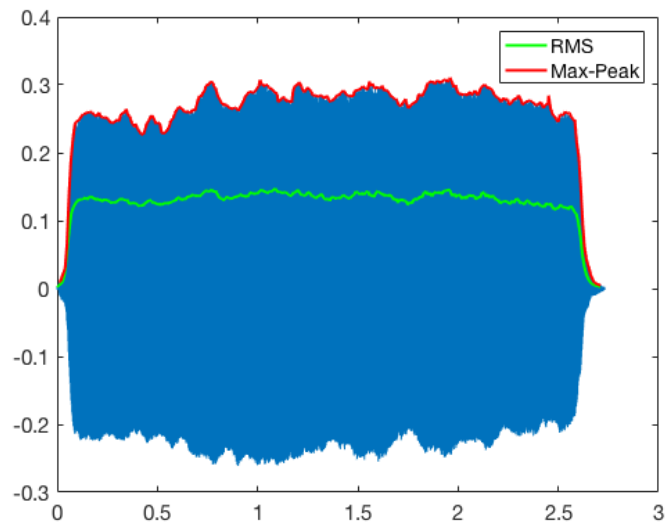


Physical Attributes in Timbre Perception

- Amplitude envelope



Piano C4 Note

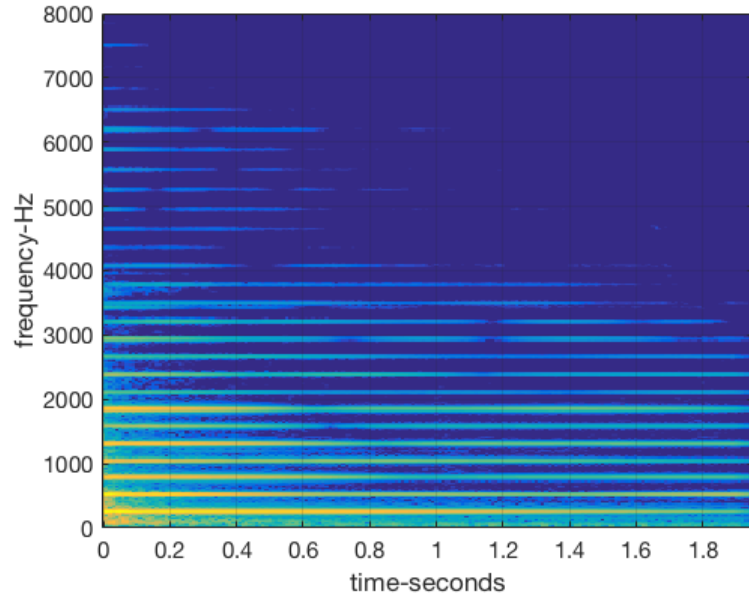


Flute Bb4 Note

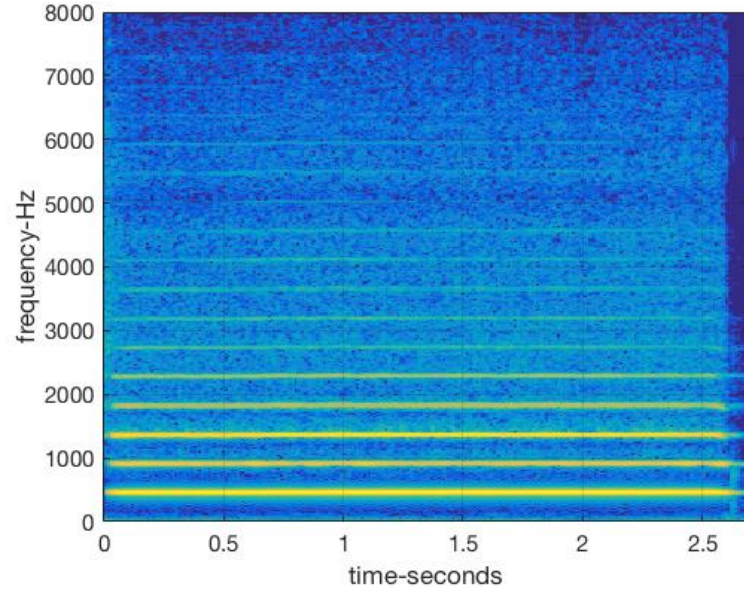


Physical Attributes in Timbre Perception

- Spectrogram



Piano C4 Note

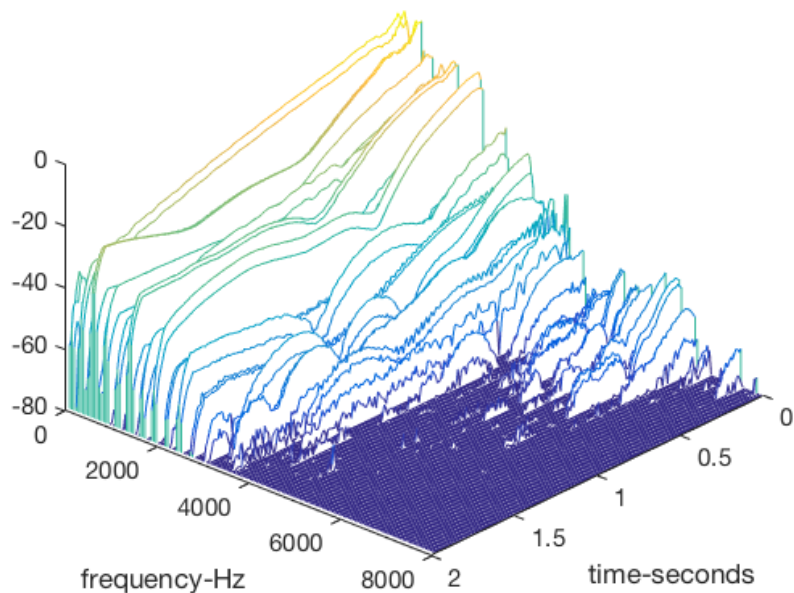


Flute Bb4 Note

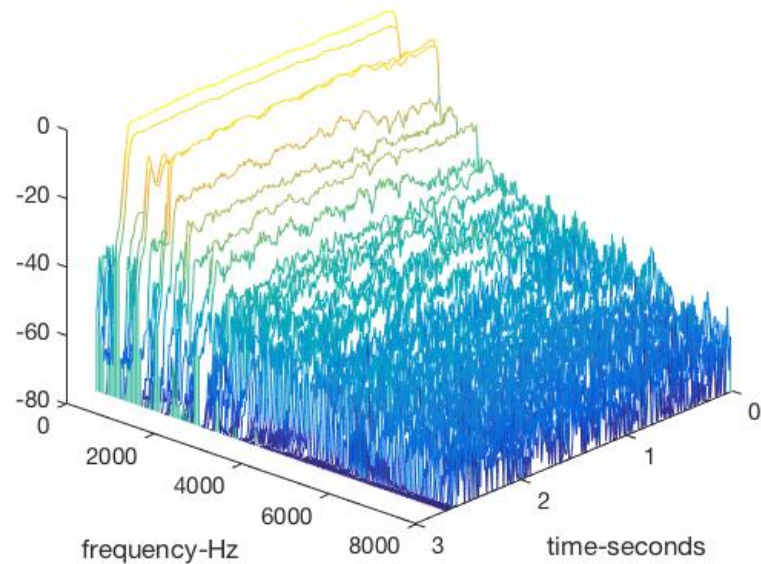


Physical Attributes in Timbre Perception

- Spectrogram – 3D Waterfall



Piano C4 Note



Flute Bb4 Note

